

Please show all work in the space provided for credit.

1.) The function $D(h) = 10e^{-0.3h}$ can be used to find the number of milligrams D of a certain drug that is in a patient's bloodstream h hours after the drug has been administered.

- How many milligrams will be present after 2 hours? _____
Round your answer to one decimal place.
- When the number of milligrams reaches 1, the drug is to be administered again.
After how many hours will the drug need to be administered? Round your answer to one decimal place.
Show work.

Cryptology is the science of making and breaking codes. This exercise explores how the idea of functions and their inverses can be used to encode and decode messages.

To encode and decode a message, first replace each letter of the alphabet with a positive integer using the following scheme, thus rewriting the original message as numbers instead of words:

A – 1	F – 6	K – 11	P – 16	U – 21	Z – 26
B – 2	G – 7	L – 12	Q – 17	V – 22	Blank - 27
C – 3	H – 8	M – 13	R – 18	W – 23	
D – 4	I – 9	N – 14	S – 19	X – 24	
E – 5	J – 10	O – 15	T – 20	Y – 25	

ENCODING: A one-to-one function can be used to encode a numerical message.

For example, suppose you want to send the message MATH to a friend, and you have decided that the function $f(x) = 3x + 4$ will be the encoding function. This function simply describes the procedure used to create the encoded message – in this case multiply by 3 and add 4. First change the letters to corresponding numbers as shown above: 13 1 20 8. Then use these as the input values in $f(x)$.

$$f(13) = 3(13) + 4 = 43$$

$$f(1) = 7$$

$$f(20) = 64$$

$$f(8) = 28$$

So the encoded message that you send to your friend is: 43 7 64 28

NOTE: A graphing calculator can be used to evaluate a function as above:

- Press [Y=] on the calculator. Type $3x + 4$.
- Choose [2nd][WINDOW] (TBLSET). Set the independent variable to Indpnt: Ask.
- Then choose [2nd][GRAPH] (TABLE). Input each x value (13 1 20 8). The calculator will return the function values (43 7 64 28) which is the encoded message.

Cryptology continued

DECODING an encoded message: Now it is up to your friend to decode the message. Decoding is the process that “undoes” the encoding process. If $f(x)$ encodes the message, what will decode it? The inverse of $f(x)$ or $f^{-1}(x)$! So $f^{-1}(x)$ will be the decoding function.

In the example, the inverse of $f(x) = 3x + 4$ can be shown to be $f^{-1}(x) = \frac{x-4}{3}$. Take the encoded message (43 7 64 28) and use these values as input values in $f^{-1}(x)$. Again, the calculator can be used to decode – simply enter $y = \frac{x-4}{3}$ and use the TABLE feature as described above.

$$\begin{aligned}f^{-1}(43) &= 13 \\f^{-1}(7) &= 1 \\f^{-1}(64) &= 20 \\f^{-1}(28) &= 8\end{aligned}$$

That’s the original numerical message! The last step is to convert back to letters using the table given previously. Now your friend knows the message that you sent: MATH.

2. Assume that the encoding function is $f(x) = 6x - 2$.

a. Find the inverse of $f(x)$. Show work.

b. Using the inverse function you just found and the letter/number chart on the previous page to decode the following encoded message. ** See calculator hint in 1st box ** The answer should be in words. Put spaces where appropriate.

16 88 70 70 28 40 28 160 4 70 40 28 10 106 4 160 52 112 160 76

148 160 34 4 130 88 106 52 118 28 160 112 124 10 58 28 16 118

Decoded message:

3.) A midwestern city had a population of 950,000 in 2004. The population, which follows the exponential model, decreased 7.63% per year. The population growth model is therefore

$$A(t) = 950000e^{-0.0763t}$$

a. If the model continues to be accurate, what will the population be in 2014?

b. When will the population be 375,000? Show work.

4.) Lynsey invests \$3000 in a bond trust that pays 8% interest compounded semiannually. She lets her money earn interest for 20 years. Her friend Lyla invests \$2500 in a certificate of deposit that pays 7.75% compounded monthly. She lets her money earn interest for 23 years. Which girl made the most money? Show use of formulas.

Which girl starts her investment with the most money? _____

Which girl has the highest interest rate? _____

Which girl has more frequent compounding for her investment ? _____

Which girl lets her money earn interest for a longer amount of time? _____

Calculate how much each girl has at the end of her investment period.

Lynsey:

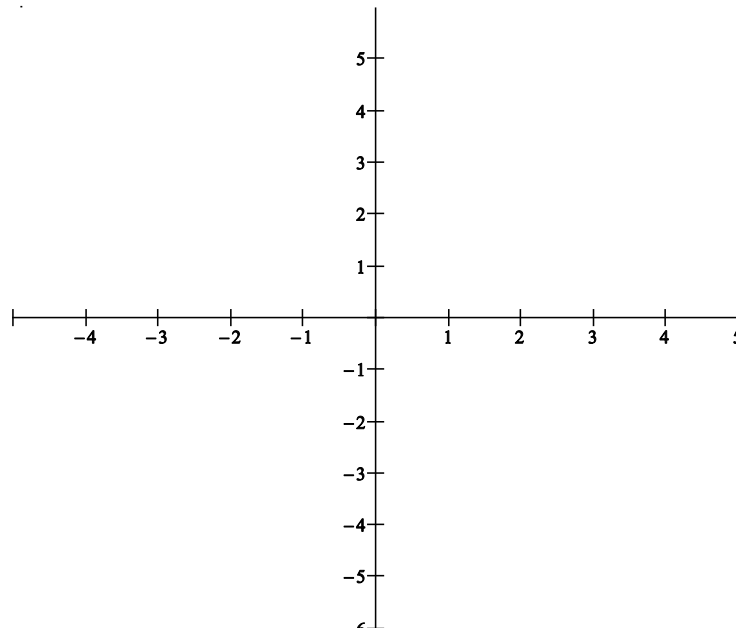
Lyla:

Which girl ended up with the larger amount of money? _____

5.) For the function $f(x) = 2^{x+1} - 3$,

a. Complete the table and use it to graph the function. Find more points if needed

x	$y = f(x) = 2^{x+1} - 3$
-2	
-1	
0	
1	
2	



b. Describe the transformations that have been applied to the graph of $f(x) = 2^x$ to obtain the graph $f(x) = 2^{x+1} - 3$.

c. What is the domain of $f(x) = 2^{x+1} - 3$? _____

d. What is the range of $f(x) = 2^{x+1} - 3$? _____

e. What are the coordinates of the y-intercept of $f(x) = 2^{x+1} - 3$? _____

f. What is the equation of the asymptote for the graph of $f(x) = 2^{x+1} - 3$? _____